

Proving the Original Definition of e

The Claim

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \quad (1)$$

Proof

$$L = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \quad (2)$$

$$\ln(L) = \lim_{n \rightarrow \infty} n \cdot \ln\left(1 + \frac{1}{n}\right) \quad (3)$$

$$= \lim_{n \rightarrow \infty} \frac{\ln\left(1 + \frac{1}{n}\right)}{\frac{1}{n}} \quad (4)$$

$$= \lim_{n \rightarrow \infty} \frac{\left(-\frac{1}{n^2}\right)}{\left(-\frac{1}{n^2}\right)} \cdot \frac{1}{1 + \frac{1}{n}} \quad (5)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} \quad (6)$$

$$= 1 \quad (7)$$

$$\therefore \ln(L) = 1 \quad (8)$$

$$\therefore L = e \quad (9)$$