Maximizing the Terms of a Sum's Product by C. D. Chester

Question:

1) You are given some number of terms n who are positive, real numbers.
2) You are also given the sum of the terms.
3) What values for the terms maximize the product?

Solution:

To attempt to solve this start simple. Say the sum of the terms is 5 and there are 2 terms. Using only integer solutions to start yields:

TERM NO. 1	TERM NO. 2	PRODUCT OF TERMS
1	4	***4***
2	3	***6***
3	2	***6***
4	1	***4***

As you can see the closer the terms get to each other the greater the product is. Thus, this yields a nifty formula for maximizing the product of terms.

Start by taking the sum and dividing it by the number of terms, effectively producing the average term of the sum.

$$\frac{sum\ of\ terms}{number\ of\ terms} = \frac{S_t}{n} = average\ term\ of\ the\ sum = A_t$$

Next, take the average term of the sum and raise it by the number of terms.

$$(A_t)^n = maximal\ product = M_p$$

Combining the two steps into one:

$$M_p = (A_t)^n = \left(\frac{S_t}{n}\right)^n$$

Applying this to the 2 terms that sum up to 5 yields a maximal product of 6.25, with each term being 2.5. By easy comparison $6.25 > 6$.

You can apply this to any relevant, related problem, given you have the number of terms and their sum.