Using the Product Rule to Obtain the Quotient Rule

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Before I begin you should have some understanding of differentiation. If you don't have any knowledge on that I suggest you understand that concept first.

Anyways the ***product rule***[1] (in Lagrange Notation[2]) is expressed as:

$$h'(x) = [f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$$

The ***quotient rule***[3] (also in Lagrange Notation) is defined as:

$$h'(x) = \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

By reforming the fraction into a multiplication problem, we can use the product rule.

<u>Proof</u>

If $i(x) = f(x)$ then $i'(x) = f'(x)$ and if $j(x) = (g(x))^{-1}$ then $j'(x) = -\big(g(x)\big)^{-2} g'(x)$ by the ***chain rule***[4].

Therefore if $h(x) = i(x)j(x)$ then $h'(x) = i'(x)j(x) + i(x)j'(x)$.

By substitution of the i and j functions (as well as their first order derivatives) you obtain: $(f'(x)[g(x)]^{-1}) - (f(x)g'(x)[g(x)]^{-2})$.

By turning $[g(x)]^{-1}$ into $[g(x)]^{-2}g(x)$ we can manipulate the equation into: $[f'(x)g(x) - f(x)g'(x)]([g(x)]^{-2})$.

Thus, giving us the same result as the prescribed quotient rule.

$\therefore Q.E.D.$

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[1] *Notation for Differentiation*
http://www.maths.manchester.ac.uk/~cds/articles/derivative.pdf

[2] Lagrange Notation is also referred to as the prime notation often denoted generally $f^n(x)$. The n in the notation is usually represented by a set number of apostrophes to represent the order of the derivative. EX: $f^1(x) = f'(x)$.
Notation: Lagrange and Leibniz
https://oregonstate.edu/instruct/mth251/cq/Stage5/Lesson/notation.html

[3] *Do We Need the Quotient Rule?*
https://ocw.mit.edu/courses/mathematics/18-01sc-single-variable-calculus-fall-2010/1.-differentiation/part-a-definition-and-basic-rules/session-11-chain-rule/MIT18_01SCF10_ex11sol.pdf

[4] For more information on this rule refer to the link below.
Chain Rule
http://mathworld.wolfram.com/ChainRule.html