Reflection About an Axis

By C. D. Chester

Imagine you have an ***axis of reflection*** that you use upon a point. For simplicity, denote the axis of reflection as $y = mx + b$ and the point as (c, d). The point after being reflected labeled as well as (e, f).

To begin this process, we must construct an ***orthogonal*** (also called perpendicular) ***line*** to the axis of reflection that goes through the point (c, d). Thus, the orthogonal line being denoted as $y = \frac{x}{-m} + g$, which then follows $g = y + \frac{x}{m} = d + \frac{c}{m} = \frac{c+dm}{m}$. Therefore, the orthogonal line, through the point (c, d), is $y = \frac{x}{-m} + \frac{c+dm}{m} = \frac{mx-c-dm}{m} = \frac{m(x-d)-c}{m}$.

Next, in finding the ***intersection of the axis of reflection and the orthogonal line***, we find half the distance necessary to reach the reflected point (e, f).

$$mx + b = \frac{m(x-d)-c}{m}$$
$$m^2x + mb = m(x-d) - c$$
$$m^2x + mb = mx - md - c$$
$$m^2x - mx = -m(b+d) - c$$
$$x(m^2 - m) = -m(b+d) - c = -(c + m(b+d))$$
$$x = \frac{-(c+m(b+d))}{m^2 - m}$$

Thus $x = \frac{-(c+m(b+d))}{m^2-m}$ is the ***x-coordinate of the intersection***, denoted x_i. Plug the x-coordinate of the intersection into the axis of reflection to achieve the ***y-coordinate of the intersection***, denoted y_i; you can also plug it into the orthogonal line if you wish.

$$y = m\left(\frac{-(c+m(b+d))}{m^2-m}\right) + b = \frac{-(c+m(b+d))}{m-1} + b$$

Thus $y = \frac{-(m)(c+m(b+d))}{m^2-m} + b$ is the y-coordinate of the intersection. Since, $\left(\frac{c+e}{2}, \frac{d+f}{c}\right)$ is the simplified format of the intersection, $\frac{c+e}{2} = x_i$, then $e = 2x_i - c$, and by a similar process, $f = 2y_i - d$. Thus, the reflected point is represented as:

$$\left(\left[\frac{-2(c+m(b+d))}{m^2-m}\right] - c, \left[\frac{-2(c+m(b+d))}{m-1}\right] + 2b - d\right)$$