

Historical Mathematics Problem: Merchants and Greed

Three merchants are traveling. They are each distinctly different in how vast their wealth is. One is poor, one is decently off, and the other is rich. Upon their travels they find a purse of gold coins. If the poor merchant takes the purse for himself, he will be two times richer than the other two merchants combined. On the other hand, if the decently off merchant obtains the purse, he will be three times richer than the other two merchants combined. Finally, if the richest merchant takes the purse, he will be five times as rich as the other two merchants combined. Given the poor merchant has only one coin, determine the amount of coins in the purse and the number of coins the other two merchants possess.

$$x = \text{poor merchant} = 1$$

$$y = \text{decently off merchant}$$

$$z = \text{rich merchant}$$

$$p = \text{purse of gold coins}$$

$$\therefore x + p = 2(y + z)$$

$$\therefore y + p = 3(x + z)$$

$$\therefore z + p = 5(x + y)$$

$$\therefore \text{If } x = 1 \text{ then } \rightarrow p = 2y + 2z - 1 = -y + 3z + 3 = 5y - z + 5$$

$$\therefore E_1 \rightarrow 2y + 2z - 1 = p$$

$$\therefore E_2 \rightarrow -y + 3z + 3 = p$$

$$\therefore E_3 \rightarrow 5y - z + 5 = p$$

$$\therefore E_1 + 2E_2 \rightarrow 8z + 5 = 3p \rightarrow z = \frac{3p - 5}{8}$$

$$\therefore \text{plugging } z \text{ into } E_3 \text{ yields } \rightarrow 5y - \left(\frac{3p - 5}{8}\right) + 5 = p \rightarrow 5y = p + \frac{3p - 5}{8} - 5$$

$$5y = \frac{8p + 3p - 5 - 40}{8} = \frac{11p - 45}{8} \rightarrow y = \frac{11p - 45}{40}$$

$$\therefore \text{plugging in } y \text{ and } z \text{ into } E_2 \text{ yields } \rightarrow -\left(\frac{11p - 45}{40}\right) + 3\left(\frac{3p - 5}{8}\right) + 3 = p$$

$$-11p + 45 + 45p - 75 + 120 = 40p \rightarrow (-11 + 45 - 40)p = -45 + 75 - 120 = -90$$

$$-6p = -90 \rightarrow p = 15$$

$$\therefore \text{If } p = 15 \text{ and } x = 1 \text{ then } y = 3 \text{ and } z = 5.$$

Therefore, the purse contains fifteen gold coins, the decently off merchant has three gold coins, and the rich merchant has five gold coins.